Quantum Music: Applying Quantum Theory to Music Theory and Composition

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QMATH 13, Georgia Tech, October 10, 2016

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Introduction: What is Music?

Music is a collection of sounds, or *tones*, that feature pitch, beat, and volume. Music consists of 3 elements:

- **Melody**: A set of musical notes, or tones, that are played in succession.
- **Harmony**: A set of musical notes that are played simultaneously.
- **Rhythm**: Duration of these sounds (or lack of sounds) in time.

Traditionally, sounds called *whole tones* are represented as follows:

For a single octave, all of the sounds used in music are represented by the *Chromatic scale*:

- Two elements of music notation that are used to denote pitch are $\flat$, the *flat* and $\sharp$, the *sharp*.
- An alternate way to represent tones is by using the number 0 to represent no sound, and the numbers $1 - 12$ as follows: $C \to 1$, $C\sharp \to 2$, $D \to 3$, ..., $A\sharp \to 11$, $B \to 12$

How to quantize Music?

A quantum tone can be represented be a quantum musical state $|\psi\rangle$. A quantum musical state would be defined as:

$$|\psi\rangle = \alpha_0 |\psi_0\rangle + \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle + \ldots + \alpha_{12} |\psi_{12}\rangle \qquad (1)$$

With $|\psi\rangle \in \mathbb{C}^{13}$ and $\sum_i |\alpha_i|^2 = 1$. Here, $|\psi_i\rangle$ are the orthogonal unit vectors:

$$|\psi_0\rangle = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, |\psi_1\rangle = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \ldots, |\psi_{12}\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

We can also define a flat and sharp operators such that:

$$\hat{\flat} \left|\psi_i\right\rangle = \psi_{i-1} \left|\psi_{i-1}\right\rangle \tag{2}$$

$$\hat{\sharp} \left|\psi_i\right\rangle = \psi_{i+1} \left|\psi_{i+1}\right\rangle \tag{3}$$

Melody: A simple example

We start by constructing a simple system of two quantum tone states

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}|\psi_0\rangle + \frac{1}{\sqrt{2}}|\psi_1\rangle \tag{4}$$

$$|\psi\rangle_2 = \frac{1}{\sqrt{2}}|\psi_0\rangle + \frac{1}{\sqrt{2}}|\psi_1\rangle \tag{5}$$

The system would have the following 4 possible outcomes, with equal probabilities:

Similarly for the following system quantum tone states,

$$|\psi\rangle_3 = \frac{1}{\sqrt{3}}|\psi_0\rangle + \frac{1}{\sqrt{3}}|\psi_1\rangle + \frac{1}{\sqrt{3}}|\psi_6\rangle \tag{6}$$

$$|\psi\rangle_4 = \frac{1}{\sqrt{3}}|\psi_0\rangle + \frac{1}{\sqrt{3}}|\psi_1\rangle + \frac{1}{\sqrt{3}}|\psi_6\rangle \tag{7}$$

The system would have the following 9 possible outcomes, with equal probabilities:

An advantage of this representation is that melodies can be composed by applying transformations, to tone states. In the previous example,

$$|\psi\rangle_4 = \hat{1}\,|\psi\rangle_3 \tag{8}$$

Harmony: Entanglement?

Quantum Harmony can be thought as an entangled state of more than one quantum note, and could be represented using quantum chords.

Challenges and Future Work

- Incorporating rhythm.
- Incorporating volume.
- Finally, how to implement all of these ideas?

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